3.3.2. The Higher Excited States of the Circle Oscillator

Now excitation $n>1$ for the energy eigenvalue equation (3.113) $\hat{H}_\omega|n,m\rangle = \hbar\omega(n+1)|n,m\rangle$.

3.3.2.1. The Possibility of the Second Excited States of the Circle Oscillator

For the double-excited state $n = n_+ + n_- = 2$, there are three cases

(3.156) $$m = n_+ - n_- = -2, 0, +2.$$

First, from (3.128) and (3.129) we write

(3.157) $$a_\pm^\dagger|1,\pm1\rangle_0 = \frac{e^{\pm i\phi}}{2}\left(\rho - \frac{\partial}{\partial\rho} \mp \frac{i}{\rho}\frac{\partial}{\partial\phi}\right)\frac{1}{\sqrt[4]{\pi}}\rho e^{-\frac{1}{2}\rho^2}e^{\pm i\phi} = \frac{1}{2}\frac{1}{\sqrt[4]{\pi}}(2\rho^2 - 1 \pm 1)e^{-\frac{1}{2}\rho^2}e^{\pm i\phi}e^{\pm i\phi},$$

then from this

(3.158) $$|2,+2\rangle = |2,0\rangle = \frac{1}{\sqrt{2}}\left(a_+^\dagger\right)^2|0,0\rangle = \quad \mathsf{P}_\pm\cdot\left(\frac{1}{\sqrt{2}\sqrt[4]{\pi}}\rho^2 e^{-\frac{1}{2}\rho^2}\right)\odot e^{+i2\phi} \quad \text{for } n=2,\ m=+2,$$

(3.159) $$|2,0\rangle = |1,1\rangle = a_+^\dagger a_-^\dagger|0,0\rangle = a_-^\dagger a_+^\dagger|0,0\rangle = \mathsf{P}_\pm\cdot\left(\frac{1}{\sqrt[4]{\pi}}(\rho^2-1)\,e^{-\frac{1}{2}\rho^2}\right)\odot \quad \text{for } n=2,\ m=0,$$

(3.160) $$|2,-2\rangle = |0,2\rangle = \frac{1}{\sqrt{2}}(a_-^\dagger)^2|0,0\rangle = \quad \mathsf{P}_\pm\cdot\left(\frac{1}{\sqrt{2}\sqrt[4]{\pi}}\rho^2 e^{-\frac{1}{2}\rho^2}\right)\odot e^{-i2\phi} \quad \text{for } n=2,\ m=-2.$$

These state expressions of radial density magnitude are shown in Figure 3.6.
As they are even functions, this contradicts Newton's 3rd law for the parity inversion balance:

$$\mathsf{P}_\pm\cdot\left(\tilde{r}_2(\rho)\right) = \tilde{r}_2(\rho) - \tilde{r}_2(-\rho) = 0 \text{ for } m=\pm2, \quad \text{or} \quad = \tilde{r}_{m=0}(\rho) - \tilde{r}_{m=0}(-\rho) = 0 \text{ for } m=0.$$

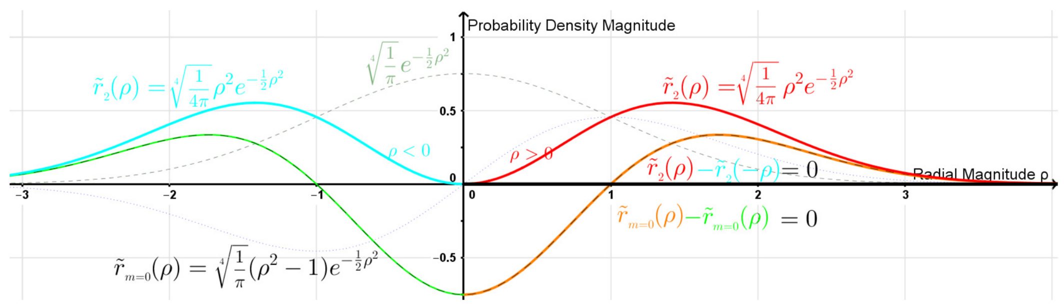

Figure 3.6 The double excitation of a single ground state provides even density functions for $n=2$; $m=-2,0,2$.
By parity difference resulting in $\mathrm{pdm}_2(\rho) = \tilde{r}_2(\rho) - \tilde{r}_2(-\rho) = 0$, for $\rho \geq 0$ as illustrated by a black line.
This is the situation for the double excitation in one and the same plane through the same event.

The even functions have self-symmetric distribution around an imagined center.
They, therefore, do not contribute as a difference over the ⊙ plane.
Therefore, the judgment is, that the double excitations in the plane of the rotation in the circle oscillation do not contribute to proper elementary ***entities*** in reality. In this way, we shall claim that two simultaneous successive operating excitations $a_\pm^\dagger a_\pm^\dagger|0,0\rangle$ in the same ⊙ transversal plane have no existence as one and the same ***entity***. This has its cause in the parity inversion balance $e^{i\pi} = -1$ along a straight line in the plane according to (3.120), which then results in a lack of contradiction and therefore disappears.
The parity inversion factor $\mathsf{P}_\pm = 0$ just displays these cancellations as characteristic of the action of the unitary $U(1)$ group of elements ⊙ ~ $e^{i\theta}$ from the plane ground state $|0,0\rangle$.
This issue of auto cancelling of double excitation $a_\pm^\dagger a_\pm^\dagger|0,0\rangle$ is limited to one and the same event location A of circulation ⊙ in one and the same plane ***direction***.

However, the creation of two or more simultaneous $a_\pm^\dagger|0,0\rangle = |1,\pm1\rangle$ from the same ground state plane ⊙ at A, are creations of multiple indistinguishable ***entities*** ${}^{\mathrm{A}}\Psi^{\odot}_{\pm\omega} \sim {}^{\mathrm{A}}|1,\pm1\rangle^{\odot}_{\omega}$.

3.3.2.2. The Possibility of a Third and Higher Excited States of the Circle Oscillator

The third excitation in the ⊙ plane attempted is written

(3.161) $$|3,\pm3\rangle_0 = \frac{1}{\sqrt{6}}\left(a_\pm^\dagger\right)^3|0,0\rangle_0 = \frac{1}{\sqrt{3}}a_\pm^\dagger|2,\pm2\rangle_0$$
$$= \frac{1}{\sqrt{6}}\frac{e^{\pm i\phi}}{2}\left(\rho - \frac{\partial}{\partial\rho} \mp \frac{i}{\rho}\frac{\partial}{\partial\phi}\right)\mathsf{P}_\pm\frac{1}{\sqrt[4]{\pi}}\rho^2 e^{-\frac{1}{2}\rho^2}e^{\pm i2\phi} = \mathsf{P}_\pm\frac{1}{\sqrt{6}}\frac{1}{\sqrt[4]{\pi}}\rho^3 e^{-\frac{1}{2}\rho^2}e^{\pm i3\phi},$$

(3.162) $$|3,\pm1\rangle_0 = \frac{1}{\sqrt{2}}a_\pm^\dagger|2,0\rangle_0$$
$$= \frac{1}{\sqrt{2}}\frac{e^{\pm i\phi}}{2}\left(\rho - \frac{\partial}{\partial\rho} \mp \frac{i}{\rho}\frac{\partial}{\partial\phi}\right)\mathsf{P}_\pm\frac{1}{\sqrt[4]{\pi}}(\rho^2-1)\,e^{-\frac{1}{2}\rho^2} = \mathsf{P}_\pm\frac{1}{\sqrt{2}}\frac{1}{\sqrt[4]{\pi}}\rho^3 e^{-\frac{1}{2}\rho^2}e^{\pm i\phi}.$$

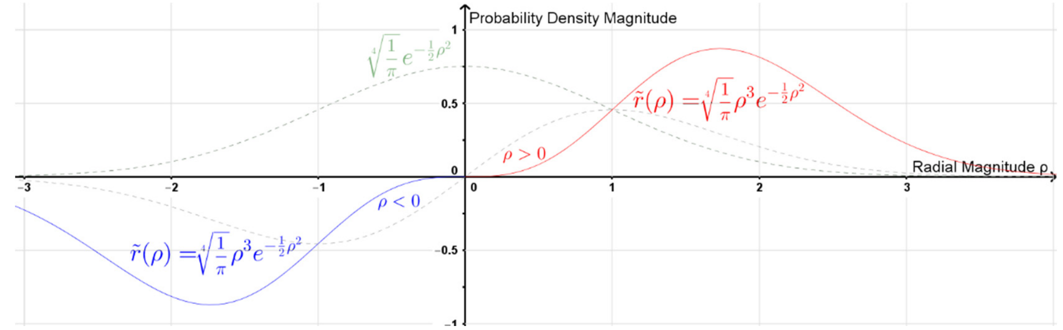

Figure 3.7 In thought tripled excitation of the ground state $|0,0\rangle$ to $|3,m\rangle$ is an odd function in one and the same plane.
(In this case without normalization factor)

Third excitations in the same plane are admittedly odd functions but build on the lack of reality of the double excitations, therefore, we assume that it will not appear in the same ⊙ plane. Since the involuting parity operation for double excitations in (3.158)-(3.160) for the same plane gives $\mathsf{P}_\pm = 0$ the even $n=2$ excitation is missing as a starting condition for additional excitations in this plane, e.g., (3.161)-(3.162) inherits $\mathsf{P}_\pm = 0$, etc.
The higher excitations can be obtained by the inclusion of several independent transversal planes (multiple dimensions) and thereby get physical reality, but we will not elaborate more on this here.[97] The multiple frequencies $n\omega$ do not occur in the circle oscillator based on multiple uses of creation operators $a_\pm^\dagger \dots a_\pm^\dagger a_\pm^\dagger|0,0\rangle$ in the same ⊙ plane.
We therefore only get the first excitations of a plane circle oscillator where $n=1$. Here all the angular frequencies $\forall\omega \in \mathbb{R}$ are permitted, as all higher multiplications $\omega = n_?\omega_?$ of some basis frequency is of course also permitted, in an idea of a circle oscillator excitation at A;
$a_{\pm\omega}^\dagger|0,0\rangle = {}^{\mathrm{A}}|1,\pm1\rangle_\omega$ for our intuition of ***entities*** ${}^{\mathrm{A}}\Psi^{\odot}_{\pm\omega} \sim {}^{\mathrm{A}}|1,\pm1\rangle^{\odot}_{\omega}$.
The only way we can distinguish these ***entities*** is by the given ***quantities*** $\omega \in \mathbb{R}$ and by the distinguishable transversal planes with ⊙ symmetries.
We remember that all $\omega' \neq \omega$ are orthogonal in thise planes.

3.3.3. The Plane Excited Circle Oscillator

In the case of light and electromagnetic waves, the concept of a transversal plane wave is already well-established in the epistemology of physics. My contention is that the circle oscillator and its symmetry $U(1)$ is an a priori intuition of the transversal plane for a propagating wave.
This review here in section 3.3 claim that we are forced only to look at the case of a first circular excitation $n=1$ of a plane for a transversal propagating wave ***entity***.

The Eigenvalue equation (3.113) demands that the angular frequency energy ω is a given ***quantity*** of the ***quality*** of excited circle oscillator creation in a plane symmetry $U(1)$ of ⊙.

[97] 20th century quantum mechanics books deal fully with the Angular Momenta in three dimensions L_x, L_y, L_z.
E.g. look in [8], [7], [9], and emeritus [29], [31], [38], alternativity [30] .