3.3. Excitation of the Plane Harmonic Circle Oscillator

3.3.1. The First Excited States of the Circle Oscillator

In particular, for the first excited state of the circle oscillator $n = n_+ + n_- = 1$ from (3.109), there are two states $m = n_+ + n_- = \pm 1$ from (3.110), that is

(3.141) $$n_+ = 1,\ n_- = 0 \ \Leftrightarrow\ n = 1,\ m = 1 \quad \text{and} \quad n_+ = 0,\ n_- = 1 \ \Leftrightarrow\ n = 1,\ m = -1.$$

The first exited state $|1_+0_-\rangle$ or $|0_+1_-\rangle$ is then by name written as

(3.142) $$|n_+n_-\rangle = |n,m\rangle \to |1,\pm 1\rangle, \qquad \text{specific as} \quad |1,+1\rangle \ \text{ or } \ |1,-1\rangle$$

One creation operation $a_\pm^\dagger$ at the ground state is $a_\pm^\dagger|0,0\rangle$ we give by the formulas (3.128)-(3.129), wherein the term $\frac{\partial}{\partial\phi}$ does not have any effect on the eternal noumenon ground state, while the terms $\frac{1}{2}\rho - \frac{1}{2}\frac{\partial}{\partial\rho}$ acting on the ground state density magnitude $\frac{1}{\sqrt[4]{\pi}}e^{-\frac{1}{2}\rho^2} \in \mathbb{R}_+$ gives

(3.143) $$\boxed{\tilde r(\rho) = \frac{1}{\sqrt[4]{\pi}}\rho e^{-\frac{1}{2}\rho^2} \in \mathbb{R},\ \text{for } \forall\rho\in\mathbb{R}}, \qquad \text{which is an odd function of the form (3.120),}$$

$\tilde r(\rho) = -\tilde r(-\rho)$, illustrated in blue $\rho<0$ and red $\rho>0$ in Figure 3.5.

$a_\pm^\dagger|0,0\rangle$

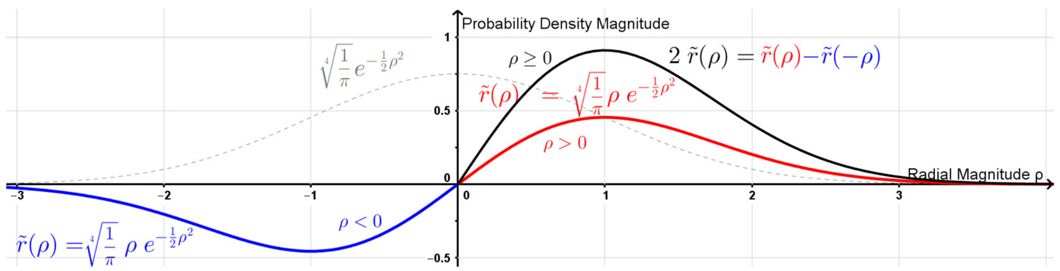

Figure 3.5 The graph magnitude for the first excited state in the circle oscillator $\tilde r(\rho) = \frac{1}{\sqrt[4]{\pi}}\rho\, e^{-\frac{1}{2}\rho^2} \in \mathbb{R}$, for $\forall\rho\in\mathbb{R}$. We see that the anti-symmetry gives a resulting difference around the center $2\,\tilde r(\rho) = \tilde r(\rho) - \tilde r(-\rho)$. This is automatically normalised as an observable object for the physical ***entity*** given $\langle 1,\pm1|1,\pm1\rangle = \langle\, |2\,\tilde r(\rho)|^2\rangle = 1$, for $\rho \geq 0$. The factor $\rho e^{-\frac{1}{2}\rho^2}$ for $\rho \geq 0$ is the Rayleigh distribution (σ=1) with the polar radial mean $\int_0^\infty \rho e^{-\frac{1}{2}\rho^2} d\rho = \sqrt{\pi/2}$.

The positive and negative density magnitude on either side of a center is in balance in accordance with the principle of Newton's third law, which is $\tilde r(\rho) + \tilde r(-\rho) = 0$.
The resulting magnitude is the objective probability ***quantity*** of the circular oscillator ***entity*** Ψ_ω is the difference between the positive and the negative density magnitude $2\,\tilde r(\rho) = \tilde r(\rho) - \tilde r(-\rho)$, thus the resulting ***quantity*** the probability density is

(3.144) $$\boxed{\mathrm{pd}_1(\rho) = 2\,\tilde r(\rho) \quad \text{for } \rho \geq 0}.$$

This probability density magnitude is only counted once for the radial coordinate $\rho \geq 0$.
The active creation operators, which act on the planar ground state rotational symmetry $\odot$ in terms of the unitary rotation group $U(1)$, may then be expressed as

(3.145) $$a_{\odot+}^\dagger = \odot 2\, a_+^\dagger = \odot e^{+i\phi}\left(\rho - \frac{\partial}{\partial\rho} - \frac{i}{\rho}\frac{\partial}{\partial\phi}\right), \quad \text{for } \rho > 0, \quad \text{and}$$

(3.146) $$a_{\odot-}^\dagger = \left(a_{\odot+}^\dagger\right)^* = \odot e^{-i\phi}\left(\rho - \frac{\partial}{\partial\rho} + \frac{i}{\rho}\frac{\partial}{\partial\phi}\right), \quad \text{these according to (3.128)-(3.129) and (3.130).}$$

The active principle requires that the rotation phase angle ϕ is a monotone variable $\phi \equiv \omega t \in \mathbb{R}$ by the information development parameter $t \in \vec{\mathbb{R}}$. The rotation intuits as an object is created from the noumenon idea of an active rotation $e^{\pm i\phi} = e^{\pm i\omega t}$, as they occur in formulas (3.145) and (3.146) for the creation operators $a_{\odot\pm}^\dagger$.

These two operators (3.145)-(3.146) commute with $\hat L_3 = \left(a_+^\dagger a_+ - a_-^\dagger a_-\right)\hbar$, hence by (3.103)

(3.147) $$\left[a_{\odot+}^\dagger, \hat L_3\right] = \left[a_{\odot-}^\dagger, \hat L_3\right] = 0.$$

By this, they have the same eigenstate as $\hat L_3$ in the eigenvalue equation $\hat L_3|n,m\rangle \doteq \hbar m|n,m\rangle$.
Therefore, the two excited states for $\rho \geq 0$; the progressive with $n = 1$, $m = +1$,

(3.148) $$\psi_+^{\odot} = |1,+1\rangle = |1,0\rangle = a_{\odot+}^\dagger|0,0\rangle = 2\left(\frac{1}{\sqrt[4]{\pi}}\rho e^{-\frac{1}{2}\rho^2}\right)\odot e^{+i\phi} = 2\tilde r(\rho)\odot e^{+i\omega t},$$

and the retrograde with $n = 1$, $m = -1$, $\updownarrow$ where $\rho \geq 0$

(3.149) $$\psi_-^{\odot} = |1,-1\rangle = |0,1\rangle = a_{\odot-}^\dagger|0,0\rangle = 2\left(\frac{1}{\sqrt[4]{\pi}}\rho e^{-\frac{1}{2}\rho^2}\right)\odot e^{-i\phi} = 2\tilde r(\rho)\odot e^{-i\omega t}.$$

It is noteworthy here, that these first excitations are direct autonomous normalised

(3.150) $$\langle 1,\pm1|1,\pm1\rangle = \int_0^\infty \psi_\pm^*(\rho)\psi_\pm(\rho)\,d\rho = \int_0^\infty \left(2\,\tilde r(\rho)\right)^2 e^{\pm i\phi}e^{\mp i\phi}d\rho = \int_0^\infty \frac{4}{\sqrt{\pi}}\rho^2 e^{-\rho^2}d\rho = 1,$$

when the ground state is prescribed $\langle 0,0|0,0\rangle = 1 \Rightarrow A_0 = \frac{1}{\sqrt[4]{\pi}}$.
I.e., that the idea $2\tilde r(\rho)$ is confirmed for $\rho \geq 0$.
When we write $\tilde r(\rho) = +\frac{1}{2}\,\mathrm{pd}_1(\rho)$ and $\tilde r(-\rho) = -\tilde r(\rho) = -\frac{1}{2}\,\mathrm{pd}_1(\rho)$,
the parity inversion contradiction can be written for $\rho \geq 0$

(3.151) $$2\tilde r(\rho) = \tilde r(\rho) - \tilde r(-\rho) = +\tfrac{1}{2}\,\mathrm{pd}_1(\rho) - \tfrac{1}{2}\,\mathrm{pd}_1(-\rho) = +\tfrac{1}{2}\,\mathrm{pd}_1(\rho) + \tfrac{1}{2}\,\mathrm{pd}_1(\rho) = \mathrm{pd}_1(\rho).$$

The excitation energy of the first state is here; according to (3.113) and (3.84),

(3.152) $$^{1}E_\omega = E_{\omega,1} - E_{\omega,0} = \hbar\omega\ .$$

By intuition look at the circular rotation described by the polar coordinates, the radial magnitude $\rho \geq 0$ and the phase angle $\phi \equiv \omega t \in \mathbb{R}$, that develops with the parameter $t \in \vec{\mathbb{R}}$.
So, we compare the classical view with the cyclical oscillator (3.117) or (3.48)
$q_\omega^*(t,r) = re^{i\omega t}$, and writes for $\theta = 0 \Rightarrow e^{i0} = 1$, for the progressive orientation

(3.153) $$q_\omega^*(t,r) = re^{+i\omega t} \sim |1,+1\rangle_{\omega,0} = |1,0\rangle_{\omega,0} = a_{\odot+}^\dagger|0,0\rangle_{\omega,0} = \left(2\frac{1}{\sqrt[4]{\pi}}\rho e^{-\frac{1}{2}\rho^2}\right)e^{+i\omega t},$$

with $n = 1$, $m = +1$; and the retrograde with $n = 1$, $m = -1$

(3.154) $$q_\omega(t,r) = re^{-i\omega t} \sim |1,-1\rangle_{\omega,0} = |0,1\rangle_{\omega,0} = a_{\odot-}^\dagger|0,0\rangle_{\omega,0} = \left(2\frac{1}{\sqrt[4]{\pi}}\rho e^{-\frac{1}{2}\rho^2}\right)e^{-i\omega t}.$$

Looking at the absolute squares we can in principle set $r = 1$, as

(3.155) $$\langle |q_\omega(t,r)|^2\rangle = r^2 \sim \langle 1,\pm1|1,\pm1\rangle = \langle\, |2\tilde r(\rho)|^2\rangle = \int_0^\infty \frac{4}{\sqrt{\pi}}\rho^2 e^{-\rho^2}d\rho = 1$$

The requirement for an ***entity*** ${}^{A}\Psi_{\pm\omega}$ in physics is when created in event A by excitation $a_\pm^\dagger|0,0\rangle$, it establishes a geometric plane $\odot = \{U_\theta : \theta \to e^{i\theta} \in U(1)\ |\ \forall\theta \in \mathbb{R}\}$ for the circle oscillator and a start phase angular ***direction*** $\phi_0 = \omega t_0$ in this plane through this event A.

The problem here is phase ***direction*** relative to what?
More on this later below in section 3.5.4 and chapter 6.

For quotation reference use: ISBN-13: 978-8797246931