(3.121) $$2\tilde{r}(\rho) = \tilde{r}(\rho) - \tilde{r}(-\rho)$$

for the oscillator ***entity*** Ψ_ω in its physics. We work here, in what I call a dipole magnitude in a plane, where there are two opposite ***quantities*** around a center, a positive magnitude $\tilde{r}(\rho)$ and a negative magnitude[87] $\tilde{r}(-\rho)$, both for $\rho \geq 0$.

3.2.4.3. Annihilation and Creation Operators in a Polar Plane

In Cartesian coordinates we describe the two rotation annihilation operators in the plane from (3.92)-(3.93) and by inserting (3.11) we get

(3.122) $$a_+ \;=\; \tfrac{1}{2}\Big(q_1 - iq_2 + \frac{\partial}{\partial q_1} - i\frac{\partial}{\partial q_2}\Big), \qquad a_- \;=\; \tfrac{1}{2}\Big(q_1 + iq_2 + \frac{\partial}{\partial q_1} + i\frac{\partial}{\partial q_2}\Big),$$

and the oscillation rotation creation operators by inserting (3.12)

(3.123) $$a_+^\dagger \;=\; \tfrac{1}{2}\Big(q_1 + iq_2 - \frac{\partial}{\partial q_1} - i\frac{\partial}{\partial q_2}\Big), \qquad a_+ \;=\; \tfrac{1}{2}\Big(q_1 - iq_2 - \frac{\partial}{\partial q_1} + i\frac{\partial}{\partial q_2}\Big).$$

Now we rewrite this in polar coordinates since we write rotation phase angles as $\phi \equiv \omega t$ and the stochastic variable radius coordinate as ρ. $(\rho^2 \leftrightarrow q_1^2 + q_2^2)$

We note the polar differential formulation for the calculation

(3.124) $$\frac{\partial}{\partial q_1} = \cos\phi\frac{\partial}{\partial \rho} - \frac{1}{\rho}\sin\phi\frac{\partial}{\partial \phi} \quad \text{and} \quad \frac{\partial}{\partial q_2} = \sin\phi\frac{\partial}{\partial \rho} + \frac{1}{\rho}\cos\phi\frac{\partial}{\partial \phi}, \quad \text{for } \forall\rho \in \mathbb{R}\backslash\{0\}$$

as by insertion (3.122) and (3.123) provides the conversion

(3.125)
$$a_\pm = \tfrac{1}{2}\Big(\rho\cos\phi \mp i\rho\sin\phi + \Big(\cos\phi\frac{\partial}{\partial\rho} - \frac{1}{\rho}\sin\phi\frac{\partial}{\partial\phi}\Big) \mp i\Big(\sin\phi\frac{\partial}{\partial\rho} + \frac{1}{\rho}\cos\phi\frac{\partial}{\partial\phi}\Big)\Big)$$
$$= \tfrac{1}{2}\Big(\rho(\cos\phi \mp i\sin\phi) + (\cos\phi \mp i\sin\phi)\frac{\partial}{\partial\rho} \mp i(\cos\phi \mp i\sin\phi)\frac{1}{\rho}\frac{\partial}{\partial\phi}\Big) = \tfrac{1}{2}e^{\mp i\phi}\Big(\rho + \frac{\partial}{\partial\rho} \mp i\frac{1}{\rho}\frac{\partial}{\partial\phi}\Big)$$
$$a_\pm^\dagger = \tfrac{1}{2}\Big(\rho\cos\phi \pm i\rho\sin\phi - \Big(\cos\phi\frac{\partial}{\partial\rho} - \frac{1}{\rho}\sin\phi\frac{\partial}{\partial\phi}\Big) \mp i\Big(\sin\phi\frac{\partial}{\partial\rho} + \frac{1}{\rho}\cos\phi\frac{\partial}{\partial\phi}\Big)\Big)$$
$$= \tfrac{1}{2}\Big(\rho(\cos\phi \pm i\sin\phi) - (\cos\phi \pm i\sin\phi)\frac{\partial}{\partial\rho} \mp i(\cos\phi \pm i\sin\phi)\frac{1}{\rho}\frac{\partial}{\partial\phi}\Big) = \tfrac{1}{2}e^{\pm i\phi}\Big(\rho - \frac{\partial}{\partial\rho} \mp i\frac{1}{\rho}\frac{\partial}{\partial\phi}\Big)$$

This results in two polar rotation oscillator annihilation operators

(3.126) $$a_+ = \frac{e^{-i\phi}}{2}\Big(\rho + \frac{\partial}{\partial\rho} - \frac{i}{\rho}\frac{\partial}{\partial\phi}\Big)$$

(3.127) $$a_- = \frac{e^{+i\phi}}{2}\Big(\rho + \frac{\partial}{\partial\rho} + \frac{i}{\rho}\frac{\partial}{\partial\phi}\Big)$$

for $\forall\rho \in \mathbb{R}\backslash\{0\}$, and correspondingly the two adjungated polar creation operators

(3.128) $$a_+^\dagger = \frac{e^{+i\phi}}{2}\Big(\rho - \frac{\partial}{\partial\rho} - \frac{i}{\rho}\frac{\partial}{\partial\phi}\Big)$$

(3.129) $$a_-^\dagger = \frac{e^{-i\phi}}{2}\Big(\rho - \frac{\partial}{\partial\rho} + \frac{i}{\rho}\frac{\partial}{\partial\phi}\Big)$$

It should be strongly noted that the adjungate $a_\pm^\dagger$ is not obtained by simple complex conjugation of $a_\pm$, but by the above calculation (3.125). By contrast, we see the rotation orientation shift

(3.130) $$a_\pm^* = a_\mp, \qquad \text{together with} \qquad \big(a_\pm^\dagger\big)^* = a_\mp^\dagger \qquad \text{by complex conjugating.}$$

Both a_+ and a_- will lower the energy state and change the angular momentum in the circle oscillator.

[87] The first to use the negative magnitude concept, in general, was Immanuel Kant in 1763. [6] p.203-27.
Negative numbers are first used by Hindus for debit and in mathematics first used by Bramagubta (ca. 628).
The term '+' and '–' symbols were introduced by the German Michael Stifel (ca. 1487-1567) who called 'the negative numbers', "numeri absurdi". This scepticism was shared by Pascal, Cardano, Newton and partly also Leibniz. They had difficulty accepting negative numbers as having physical significance. Even here, where they are included in the balance; the resulting probability is not negative. But with e.g., acceptance of negative charges, the negative numbers do not cause problems in modern physics.

3.2.5. Ground State of the Circle Oscillator

For the ground state in (3.115) $E_{\omega,0} = \hbar\omega$, where $n=0 \Rightarrow m=0$ for $\widehat{H}_\omega$ (3.113), we must apply

(3.131) $$a_+|0{,}0\rangle \;=\; a_-|0{,}0\rangle \;=0$$

Both the real and imaginary parts of the operators a_+ and a_- shall fulfil this condition, therefore

(3.132) $$\Big(\rho + \frac{\partial}{\partial\rho}\Big)|0{,}0\rangle = 0$$

(3.133) $$\frac{\partial}{\partial\phi}|0{,}0\rangle = 0$$

The differential equation (3.132) like (3.42) and (3.43) has a real solution (3.44)

(3.134) $$|0{,}0\rangle_0 = A_0 e^{-\frac{1}{2}\rho^2} \geq 0$$

While the last condition (3.133) is the important impact that the ground state idea is constant regardless of the angle of rotation $\phi = \omega t$. Any arbitrary complex factor $e^{i\theta}|0{,}0\rangle_0$, where $|e^{i\theta}|=1$ to the initial state $|0{,}0\rangle_0$ make no change, corresponding to a stochastic angle $\theta \in [0{,}2\pi[$ as a ***direction*** in the plane for the ground state, that is eternally unchangeable, i.e. $\frac{\partial}{\partial\phi}\theta = 0$ for $\forall\theta$.

The random factor $e^{i\theta}$ in the ground state $|0{,}0\rangle = e^{i\theta}|0{,}0\rangle_0$ make it rotation symmetric invariant through any arbitrary plane for the ground state idea.

Claim a judgment: Any arbitrary point is the center of its own ground state; a point per se.

The ground state, according to (3.133) is self-identical for all $\phi = \omega t$ and a factor $e^{i\theta}$ is giving the rotation invariant identity through a plane. We incorporate this uncertainty factor with the symbol

(3.135) $$\odot \sim e^{i\theta} \quad \text{for } \forall\theta \in [0{,}2\pi[$$

as in $A = A_0 e^{i\theta} \sim \odot A_0$, and get the full solution to (3.132) and (3.133) in the rotation plan

(3.136) $$|0{,}0\rangle \;=\; Ae^{-\frac{1}{2}\rho^2} \;=\; A_0 e^{i\theta}e^{-\frac{1}{2}\rho^2} \;\sim\; \odot A_0 e^{-\frac{1}{2}\rho^2} \;\in \mathbb{C},$$

Wherein the invariance is given by $\frac{\partial}{\partial\phi}\theta = \frac{\partial}{\partial\phi}\odot = 0$ to all outward ***directions*** from a center in an arbitrary plane.[88] The rotational symmetry of the concept of a plane through the unitary rotation group $U(1)$ is expressed by the symbol

(3.137) $$\odot = \{U_\theta : \theta \to e^{i\theta} \in U(1) \mid \forall\theta \in \mathbb{R}\}.$$

For all the polar coordinates (ρ, θ) we have $\rho^2 = (-\rho)^2 \geq 0$, therefore $|0{,}0\rangle = \odot A_0 e^{-\frac{1}{2}\rho^2}$ is parity inversion indifferent.[89] It makes no difference by ρ and $-\rho$, therefore, the ground state is an even function (and indifferent). The ground state is eternal and independent, not only in relation to all relationships but also to the concept of ***direction***.[90]

The normalization of the $\langle 0{,}0|0{,}0\rangle = 1 \;\Rightarrow\; A_0 = \frac{1}{\sqrt[4]{\pi}}$, in that $|e^{i\theta}| = |\odot| = 1$.

The real scalar density magnitude for the ground state, given at an arbitrary stochastic radius coordinate magnitude ρ, is

(3.138) $$\big||0{,}0,\rangle_{(\rho)}\big| \;=\; \mathrm{pdm}_0(\rho) \;=\; \frac{1}{\sqrt[4]{\pi}}e^{-\frac{1}{2}\rho^2} \;\in \mathbb{R}_+,$$

that's equivalent to (3.134) and (3.44) and showed in Figure 3.3, for which the implicit ***direction*** $\theta \in [0{,}2\pi[$ is random arbitrary through the plane for intuition of a virtual rotation.

[88] The factor $\odot$ is simply unitary complex numbers that commute with the other factors in expressions, so all placements are valid.
[89] The factor $e^{i\pi} = -1$ does not make any difference to the ground state because it is constant $\frac{\partial}{\partial\phi}e^{i\theta} = 0$ and $|e^{i\theta}| = |e^{i\pi}| = 1$.
[90] The concept of ***direction*** in space will be described in further detail in the chapter Geometry of Physics p.123→II. 6, by Geometric Algebra. Immanuel Kant was the first to problematise this in 1768 [11]p.361.