The axis of rotation is defined by a local unit vector $\hat{\vec{\omega}} \equiv \vec{n}$ as a normal vector to the circle plane. We define a rotation vector $\vec{\omega}$ for the oscillator circular motion

(3.54) $\vec{\omega} = \frac{\partial(\omega t)}{\partial t}\hat{\vec{\omega}}$

The circle oscillation takes place in transversal plane to an axis of rotation $\vec{\omega} = \omega\vec{n}$.

$\vec{\omega}$ are linearly independent of $\vec{q} \sim (q_1, q_2)$ and $\dot{\vec{q}} \sim (\dot{q}_1, \dot{q}_2)$.

The rotation vector is orthogonal to both $\vec{q}$ and $\dot{\vec{q}}$,

I.e., that[73] $\vec{\omega} \cdot \vec{q} = 0$ and $\vec{\omega} \cdot \dot{\vec{q}} = 0$.

Geometric expressed it is as $\vec{\omega} \perp \vec{q}$ and $\vec{\omega} \perp \dot{\vec{q}}$, or $\vec{\omega} \perp \vec{r}$ and $\vec{\omega} \perp \vec{v}$, as shown in Figure 3.2.

I.e., the axis of rotation $\vec{\omega}$ is perpendicular to the circle plane, or algebraic expressed as orthogonal to this.

We say that the circle plane is transversal to $\vec{\omega}$.

In three dimensions, we apply vector cross product

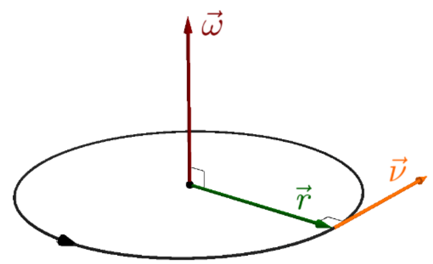

Figure 3.2 The ***quantity*** of $\vec{\omega}$ of the circle oscillator, where $\vec{v} \perp \vec{r}$, $\vec{\omega} \perp \vec{r}$ and $\vec{\omega} \perp \vec{v}$

(3.55) $\vec{\omega} = \frac{\vec{r} \times \vec{v}}{|\vec{r}|^2}$, note for later use[74],

and the rotation is written in axis-coordinate

(3.55a) $\omega_3 = +\frac{(q_1\dot{q}_2 - q_2\dot{q}_1)}{r^2}$, (that anti commute 1,2↔2,1 ⇒ ±, therefor $\vec{\omega}$ is a pseudo-vector)

whereby the circle-radius and circle-velocity here are expressed in Cartesian coordinates $\mathbb{R}^3_\perp$

(3.56) $\vec{r} = \vec{q} \sim (q_1, q_2, 0) \in \mathbb{R}^3_\perp$ and $\vec{v} = \vec{v}_{\perp r} = \dot{\vec{q}} \sim (\dot{q}_1, \dot{q}_2, 0) \in \mathbb{R}^3_\perp$,

and the axis of rotation $\vec{\omega} = \omega_3 \cdot \hat{\vec{\omega}} \sim \omega_3 \cdot (0,0,1)$ or simply $\vec{\omega} \sim (0,0,\omega_3)$.

The coordinate expression of the cross product (3.55a) can be rewritten by (3.48)-(3.49) to

(3.57)
$$\omega_3 = (q_1\dot{q}_2 - q_2\dot{q}_1)/r^2 = \cos(\omega t)\cdot\omega\cos(\omega t) + \sin(\omega t)\cdot\omega\sin(\omega t)$$
$$= \omega\cos^2(\omega t) + \omega\sin^2(\omega t) = \omega.$$

The reader needs to note, that ω is independent of the radius r of the circle.

We define from (3.48) the unit circular oscillating progressive rotation in a plane as

(3.58) $u^*_\omega = q^*_\omega(1,t) = 1e^{i\omega t} = e^{i\omega t}$,

when we concern $\omega > 0$ as positive real, and thus define the reverse unit retrograde as

(3.58a) $u_\omega = e^{-i\omega t}$

3.1.7.2. The Transversal Plane

When you are looking at the circular rotation plane by intuition, we define the ***direction*** of the normal $\hat{\vec{\omega}} \equiv \vec{n}$ vector to the plane as *FORWARD*, i.e., when we look at the circle plane ⊙ (e.g., the paper plane exists for the intuition) by receiving light into your eye through the plane.

A plane defined by a ***directional*** 1-vector[75] $\hat{\vec{\omega}} \equiv \vec{n}$ is called a transversal plane to that normal 1-vector ***direction*** and perpendicular to it. We remember by nomination $|\hat{\vec{\omega}}| \equiv 1$.

Do we have a rotation given by a local vector $\vec{\omega}$, the corresponding normalized 1-vector $\hat{\vec{\omega}} = \vec{n} = \pm\vec{\omega}/|\vec{\omega}|$ for the *FORWARD* ***direction*** has two stats depending on whether the rotation vector $\vec{\omega}$ represents a progressive (3.58) or a retrograde rotation oscillation.

[73] The scalar product of orthogonal vectors is 0 by definition. This we always apply to perpendicular intuit object vectors.

[74] In Chapter 6 we write this in the geometric algebra as a bivector $\mathbf{B} = (\vec{r} \wedge \vec{v})/|\vec{r}|^2$. This will be connected with by a so-called pseudoscalar unit $\boldsymbol{i}$, ($\boldsymbol{i}^2 = -1$ and $\boldsymbol{i}^{-1} = -\boldsymbol{i}$), that multiplied gives the bivector $\mathbf{B} = \mathbf{\Omega} = \boldsymbol{i}\vec{\omega}$. Conversely, we will call the rotation axis described here a pseudo-vector $\vec{\omega} = -\boldsymbol{i}\,\mathbf{B} = -\boldsymbol{i}(\vec{r} \wedge \vec{v})/|\vec{r}|^2$ just as the Gips cross-product standard .

[75] For now, a 1-vector can be interpreted as an arrow in space by intuition, later it is a special case of the multi-vectors concept.

Anyway, the ***direction*** of the transversal plane itself[76] has a positive orientation around the progressive rotation. Has $\vec{\omega} = -|\omega|\hat{\vec{\omega}}$ a negative orientation relative to the intuitive *FORWARD* normal $\hat{\vec{\omega}} \equiv \vec{n}$ ***direction***, the circular oscillating rotation in the transversal plane is retrograde.
In physics, we generally accept $\omega \in \mathbb{R}$ to be both positive and negative, and therefore circular oscillation that leads to four cases:

rotation	$u_\omega = e^{-i\omega t}$	$u^*_\omega = e^{+i\omega t}$	↻ ↺
$\omega > 0$	retrograde – clockwise	progressive - contra-clock	b d
$\omega \leq 0$	progressive – contra-clock	retrograde - clockwise	p q

This symmetry is the same as expressed in the four letters: b d p q that all have the same form b in space but appear ambiguous and different to the looking eye in four ways.
The view on these four form similar letters shall be transversal to a perpendicular look.
If b or q is seen from the front, p or d is seen from the back, for the same ***entity***.

The progressive oscillator rotation $e^{i\omega t}$ causes ω positive, when we look in the tip ⊙ of $\hat{\vec{\omega}}$ (right-hand rule), and negative[77] when we see it from behind ⊗. We have $\vec{\omega} = \omega\hat{\vec{\omega}}$.
Anyway, be careful with the sign of rotation orientation. The positive orientated ***direction*** of

(3.59) $\hat{\vec{\omega}} \equiv \vec{n} = \hat{\mathbf{1}}$

is the *FORWARD* ***direction*** of light trough, from or by the transversal plane. (The plane wave concept)
Hope you got the standard, of this view. If you think, it is weird reconsider. It is quite simple and especially important for the ethical understanding of the foundation of physics.

3.1.7.3. The Rotating Direction with Orientation

The angular frequency ω is the oriented length of the rotation vector $\vec{\omega}$, with $|\omega| = |\vec{\omega}|$ as the magnitude of the rotation vector. The sign, $\text{sign}(\omega) = \pm 1$ indicates an orientation of the rotation $\circlearrowleft^+$or $\circlearrowright^-$ relative to orientation of the transversal plane in agreement with the $\hat{\vec{\omega}}$ ***direction***.
The angular frequency ω is a given ***quantity*** of the circle oscillator.

The possibility of a circle rotation in the plane with a fundamental ***direction*** $\hat{\vec{\omega}}$, we call a ***primary quality*** of the substance of circle oscillation.

We note that a given vector $\vec{\omega} = \omega \cdot \hat{\vec{\omega}}$ with a given specific ***direction*** $\hat{\vec{\omega}}$ and given ***quantity*** ω can be an intuition object - the *harmonic circle oscillator*.[78]
Later, we will look more into the chiral aspect through or from the circle rotation plane, and thus the pseudo-vector aspect of the vector $\vec{\omega} = \omega \cdot \hat{\vec{\omega}}$.

3.1.7.4. An Idea of a Primary Quantum Operator

In the perspective of the concept of a circle oscillation, from the form $\vec{\omega} = \omega \cdot \hat{\vec{\omega}}$ we define an operator $\hat{\omega} \coloneqq \omega\,\hat{\mathbf{1}}$, wherein the unit operator for a given ***direction*** $\hat{\mathbf{1}}$ is a ***primary symmetry quality***. (More about spatial ***direction*** and symmetry later below.)

We emphasise that the given ***quantity*** ω described as an operator in that ***direction*** applies

(3.60) $\hat{\omega}(\varphi) \coloneqq i\frac{\partial}{\partial t}(\varphi)$ and $[\hat{t}, \hat{\omega}] = i$. $\hbar = 1$

To understanding this, I ask the reader to investigate the formulas (2.40), (2.73) and (2.78)-(2.80), and compare with the Schrödinger equation (2.65)

(3.61) $i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle$, where $\hat{H} \sim \hbar\hat{\omega}$.

[76] The ***direction*** of a plane will later be defined by the concept of a bivector (bi-vector) in the geometric algebra, e.g., ***i***, § 5.2.5.

[77] ω is a scalar quantity of the circle and $\vec{\omega}$ is a pseudo-vector for rotation, therefore $|\vec{\omega}| = |\omega|$.
These pseudo objects just change sign by the 3D view angle (viewing direction, as b d p q), more about this later.

[78] We note that the magnitude of the circle radius $|\vec{r}|$ is irrelevant to the circle rotation ***quantity*** ω as a ***primary quality***.

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