

Similar to (1.82), (1.83) and (1.87), (1.88) we include the useful formulations

$$(2.85) \quad \langle p_j | \hat{q}_j | p_j' \rangle = i \frac{\partial}{\partial p_j} \cdot \delta(p_j - p_j'),$$

$$(2.86) \quad \langle q_j | \hat{p}_j | q_j' \rangle = -i \frac{\partial}{\partial q_j} \cdot \delta(q_j - q_j').$$

Here δ is the Dirac delta function.

2.2.7.2. The Measurable Expectation Values of Quantum Mechanics

For a physical *entity* Ψ with observable or at least identifiable *quantities* q_j , with canonical conjugated *momenta* p_j , we must expect different probabilities

$\psi_{q_j} = \psi(q_j)$ and $\psi_{p_j} = \psi(p_j)$ to measure the different variations in their values.

Most often the probability functions are described abstract as ψ , where the dependency argument input is omitted, but these must be implicit given the context.

A real *quantity* q_j represented by the Hermitian operator \hat{q}_j can then act on the associated probability function ψ , $\hat{q}_j |\psi\rangle$. Just as defined in (2.45) we write the expectation value of the observable *quantity* for the operator \hat{q}_j

$$(2.87) \quad \langle \hat{q}_j \rangle_\psi = \langle \psi | \hat{q}_j | \psi \rangle = \langle \psi | \hat{q}_j \psi \rangle \sim \int_{-\infty}^{\infty} \psi^*(y) \hat{q}_j \psi(y) dy \sim \int_{-\infty}^{\infty} q_j |\psi|^2 dq_j.$$

Generally, we just write the expectation value of \hat{q}_j as $\langle \psi | \hat{q}_j | \psi \rangle$.

Similarly, writing the observable expectation value of \hat{p}_j as $\langle \psi | \hat{p}_j | \psi \rangle$.

2.3. A classical Formulation of the Cyclic Rotation Oscillation

2.3.1. Hamilton Formulation for the Harmonic Oscillator

In section 1.7.6 for the oscillator in physics, we got formulas for \dot{q}_ω (1.76) and \dot{p}_ω (1.79).

Inserting these in Hamilton's canonical equations (2.21) and (2.22) we get

$$(2.88) \quad \dot{q}_\omega = \frac{\partial H_\omega}{\partial p_\omega} = \frac{p_\omega}{m_\omega}$$

$$(2.89) \quad \dot{p}_\omega = -\frac{\partial H_\omega}{\partial q_\omega} = -m_\omega \omega^2 q_\omega$$

Now we can create the simplest possible Hamilton function for the harmonic oscillator.

From the stationary differential (2.20) $dH = \dot{q} dp - \dot{p} dq$, and rewriting by (1.76) to

$$(2.90) \quad dH_\omega = \dot{q}_\omega dp_\omega + m_\omega \omega^2 q_\omega dq_\omega,$$

we get by antiderivative integration

$$(2.91) \quad H_\omega(q_\omega, p_\omega) = \frac{1}{2} p_\omega \dot{q}_\omega + \frac{1}{2} m_\omega \omega^2 q_\omega^2 = \frac{p_\omega^2}{2 m_\omega} + \frac{1}{2} m_\omega \omega^2 q_\omega^2.$$

To make this equation reliable, we must from the definition require $p_\omega^2 \equiv p_\omega m \dot{q}_\omega$.

The last form has the advantage that the parameter t is not included. But then there is a minor problem, the first term in the sum H_ω of this formula is undefined if $m_\omega = 0$.⁶⁶

⁶⁶ 0/0 need not be 0, or neither a constant without change, but an arbitrary variable with all values simultaneously. This is solved by the fact that $m_\omega \neq 0$ is a factor in the *internal* impedance of the oscillator, and not the *external* mass.

2.3.1.2. The Energy of an Oscillator

Classic, the first part in the sum has been attributed to the kinetic energy T , and the second part has been attributed to the potential energy V , so $H = T + V$, whereby

$$(2.92) \quad T_\omega = \frac{1}{2} p_\omega \dot{q}_\omega = \frac{(p_\omega)^2}{2 m_\omega} \quad \text{and} \quad V_\omega = \frac{1}{2} m_\omega \omega^2 q_\omega^2, \quad \text{hence} \quad H_\omega = T_\omega + V_\omega.$$

Since the first part is written $T_\omega = \frac{1}{2} p_\omega \dot{q}_\omega$, we from (2.17) conclude $H_\omega = 2T_\omega - L_\omega$

and from which we draw the *portable energy*

$$(2.93) \quad L_\omega = T_\omega - V_\omega$$

Hereby, L_ω represent the difference between kinetic and potential energy.

Stated differently, the portable energy L_ω of an *entity* Ψ_ω in physics consists of the free kinetic energy T_ω , minus the energy V_ω of the bindings to some fundamental surroundings.

We will distinguish between; the internal bindings of the oscillator *entity* Ψ_ω , that has a proportionality factor we call m_ω ; and an external binding energy to the surroundings V_Ψ of a total physical *entity* Ψ characterised by a proportionality factor m , that in classical physics is called the mass of Ψ . It is important in the following to distinguish between the internal factor m_ω of the oscillator, from any external m , for a total physical *entity* Ψ .

2.3.1.3. The Lagrange Function for the Cyclical Rotating Oscillator

By using $T_\omega = \frac{1}{2} \dot{q}_\omega p_\omega = \frac{1}{2} m_\omega \dot{q}_\omega^2$ for the first part, we can now rewrite (2.93) to

$$(2.94) \quad L_\omega(q_\omega, \dot{q}_\omega) = \frac{1}{2} (\dot{q}_\omega^2 - \omega^2 q_\omega^2).$$

The easy way to set $L_\omega = 0$ e.g. for a photon is to set $m_\omega = 0$, but then we lose the knowledge of the structure for the oscillator, on the other hand, the inner cyclical balance can be maintained by demanding $\frac{1}{2} (\dot{q}_\omega^2 - \omega^2 q_\omega^2) = 0$, hence we achieve that $L_\omega(q, \dot{q}) = 0$, independent of $m_\omega \neq 0$.

When both parts \dot{q}_ω^2 and $\omega^2 q_\omega^2$ are constant and equal it is a circular oscillation. Thus, the *entity* $\Psi_\omega \leftrightarrow \frac{1}{2} ((\dot{q}_\omega)^2 - \omega^2 q_\omega^2) = 0$ as a subject is interpreted as a cyclic oscillation through the complex unit circle $e^{\pm i\omega t}$ as an object to our intuition.

Its noumenon (idea in science) is the Euler circle as a *primary quality* of a plane idea in physics.

Another way of expressing $L_\omega(q, \dot{q}) = 0$ is simply by letting $T_\omega - V_\omega = 0$ in (2.93).

To let the Lagrange function disappear, $L_\omega = 0$ for a harmonic oscillator is the same as to claim, that it doesn't carry any portable energy through the surroundings in physics, but that the internal binding energy V_ω balance the kinetic energy T_ω , that is

$$(2.95) \quad L_\omega = T_\omega - V_\omega = 0.$$

When the harmonic oscillator has non-external binding $V_\omega^{extern} = 0$ to the surroundings, it will move freely through the surroundings with kinetic energy T_ω .

In this way, the free harmonic oscillator Ψ_ω has the total energy as the Hamilton function

$$(2.96) \quad H_\omega = T_\omega + V_\omega = 2T_\omega, \quad \text{in that} \quad L_\omega = T_\omega - V_\omega = 0.$$

This gives the idea of the nature of light, through electromagnetic oscillation, which will now be described by employing of the *quantum harmonic oscillator* joint by the noumenon idea of a rotating circle oscillation in a *quantum field*.