2.2.6. The Frequency-energy Quantised Evolution Parameter

In addition to appointing the given ***quantity*** ω for the oscillator to be an operator $\widehat{\omega}$,
I also promote the development parameter $t \in \vec{\mathbb{R}}$ to correspond with an evolution ***quantity*** $q_0 = q_0(t) \sim c{\cdot}t$, not only for the individual oscillator, but for all the oscillators in the local physical ***entity*** Ψ.
Is the measure of q_0 in the same unit as t, e.g. [s], you can apply $c = \pm 1$, thus $q_0 \sim \pm t$.
For each oscillator Ψ_ω we now define an expression for an energy mode operator

(2.73) $$\hat{p}_0 = \hat{p}_0(\omega) \coloneqq \widehat{\omega} = i\frac{\partial}{\partial t} = i\frac{\hbar}{c}\frac{\partial}{\partial q_0}. \qquad (\hbar{=}1)$$

Now I will appoint $\hat{q}_0 \sim c{\cdot}t$ to a parameter operator, or seen as an evolutionary operator.[60]
To increase understanding we can write $\hat{q}_0 = c{\cdot}t{\cdot}\hat{1}_0$ or $\hat{q}_0 = q_0{\cdot}\hat{1}_0$, where $\hat{1}_0$ is a desired unit vector ***direction*** for the counting operator $+1$.[61]
We demand that $\hat{q}_0, \hat{p}_0$ are canonical conjugated. From (2.71) we write

(2.74) $$[\hat{q}_0, \hat{p}_0] = i. \qquad (\hbar{=}1)$$

We now take note that we can just write (2.52) as the Heisenberg picture

(2.75) $$\frac{d}{dt}\hat{F} = -i\left[\hat{F}, \widehat{\omega}\right]$$

In addition, we note the pure mathematical approach of the operator commutator

(2.76) $$\left[\hat{F},\ i\frac{\partial}{\partial t}\right] = -i\frac{\partial \hat{F}}{\partial t}, \quad \text{since} \quad \left[\hat{F}, i\frac{\partial}{\partial t}\right]\psi = \hat{F}\, i\frac{\partial}{\partial t}\psi - i\frac{\partial}{\partial t}\left(\hat{F}\psi\right) = \hat{F}\, i\frac{\partial}{\partial t}\psi - \hat{F}\, i\frac{\partial}{\partial t}\psi - i\frac{\partial \hat{F}}{\partial t}(\psi) = -\,i\frac{\partial \hat{F}}{\partial t}\psi.$$

Inserting the definition of $\hat{q}_0$ in this, we get

(2.77) $$\left[\hat{q}_0, i\frac{\partial}{\partial t}\right] = -i\frac{\partial \hat{q}_0}{\partial t} = -i{\cdot}c$$

If we compare this through the definition of $\hat{p}_0 \coloneqq i\frac{\partial}{\partial t}$ with (2.74), we get that c is negative.
Do we fix[62] $c = -1$, and thus $q_0 \coloneqq -t$ by definition, we can write

(2.78) $$\left[\hat{q}_0, i\frac{\partial}{\partial q_0}\right] = i, \qquad \text{we can also write} \qquad \left[\hat{t}, i\frac{\partial}{\partial t}\right] = i$$

The parameter operator $\hat{t} = t{\cdot}\hat{1}_0$ can of cause only be interpreted as giving a monotone positive growing real number $t \in \vec{\mathbb{R}}$. The question here: Is $\hat{t}$ a time-ordering operator ***direction*** *FORWARD*, but without a causal initiative?[63]
Simple from (2.73) we can write the commutator between the two canonical conjugated

(2.79) $$[\hat{t}, \widehat{\omega}] = i \qquad \hbar{=}1$$

This relationship implies the uncertainty principle. When the energy is given as a conserved specific angular frequency ω of the oscillator, then the parameter t is undetermined.
Which means that t not imply any causal relationship.
We repeat here, that the product $\omega t \in \mathbb{R}$ is simply the angle of rotation in a circle oscillator Ψ_ω.
From here we summarise that if p_0 is measured with the same unit as ω $(\hbar{=}1)$, and q_0 is measured in the same units as t $(c = -1)$, we can write a basic a priori law of physics.[64]

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[60] Some might think of calling it the time operator, which can cause problems in understanding.
[61] Later in chapter II. 5.7 and III. 7 we call $\hat{1}_0 = \gamma_0$, where we will count $\gamma_0^2 = 1 \Rightarrow |\gamma_0| = |\hat{1}_0| = 1$.
[62] This can be done, when the ***quantity*** q_0 and the parameter t are measured by the same measure.
[63] Anyway; I claim my synthetic judgement, that the concept of time parameter t as a real scalar never can be a pseudo scalar, $\hat{t}^2 = \pm|\hat{t}|^2$, because it is born as synchronous count by causality from a 'tuck' from a clock, by linear multiplication of a transversal plane pseudoscalar bivector $\boldsymbol{i}$ as the generating argument in a unitary circular exponential function.
You are invited to contradict me? But first, you should read the rest of this book.
[64] That $q_0 = -t$ is the most fundamental measure in physics given by the canonical adjoined p_0. Things must wait for information from other things (finite speed c). This will be discussed later. Compare first with (1.84) in section 1.7.8.

(2.80) $$\boxed{\hat{p}_0 \coloneqq -i\frac{\partial}{\partial q_0}} \quad \text{in that,} \quad \hat{p}_0 \coloneqq \hbar\widehat{\omega} = i\hbar\frac{\partial}{\partial t} = i\frac{\hbar}{c}\frac{\partial}{\partial q_0}, \qquad (c{=}-1,\ \hbar{=}1).$$

The relationship becomes even simpler when we use the reciprocal units:
E.g., for $p_0\ [s^{-1}] \sim$ [per second] and for $q_0\ [s] \sim$ [second].
We note now the fundamental law of physics; that the ***quantity*** q_0 with the corresponding canonical conjugate ***quantity*** p_0 is not only mutually orthogonal, but also meet $[\hat{q}_0, \hat{p}_0] = i$ for the two corresponding operators $\hat{q}$ and $\hat{p}$ for the arbitrary local ***entities*** Ψ in physics.
The scalar product of these two resulting ***quantities*** is the phase angle $\phi = p_0{\cdot}q_0 = -\omega t$ of the ***primary quality*** of a rotating **circle oscillation** (2.37) $q_\omega(t) = \tilde{q}_\omega e^{-i\omega t}$.

2.2.7. Momentum

We have in this chapter concerning the concept of time versus the development parameter introduced the cyclical oscillator as the fundamental cause. The clock, as a cyclical oscillator is the foundation that determines the *FORWARD* ***quality*** of the one development parameter $t \in \vec{\mathbb{R}}$, and thus the frequency ratios between all the oscillators in some local ***entity*** Ψ in physics.
We have tried to introduce abstract ***quantities*** q_ω and $\dot{q}_\omega$ of the oscillator (1.60) and (1.61) which are mutually orthogonal. We also introduced (1.76) an internal momentum ***quantity*** p_ω of the oscillator. With the Lagrange formalism we had generalised the ***quantities*** q and $\dot{q}$. Further with the Hamilton formalism we replaced $\dot{q}$ with p, where p is the **canonical conjugate momentum** to the generalised ***quantity*** q.
Now we look at a generalised ***entity*** Ψ defined by $N{+}1$ ***quantities*** q_j, for $j = 0,1,2, \dots N$.

2.2.7.1. The Canonical Quantum Operators

From here, when we define the ***quantum momentum operators*** $\hat{p}_j$ with the **canonical conjugated *quantity operators*** $\hat{q}_j$, where $j = 0,1,2, \dots N \in \mathbb{N}$, we must require that they comply with (2.71), so that the following general relationship applies in accordance with (2.73) and (2.80) assuming that $\hbar = 1$, $c = -1$;

(2.81) $$\hat{p}_j = -i\frac{\partial}{\partial q_j} \quad \text{and} \quad \hat{q}_j = i\frac{\partial}{\partial p_j}.$$

Now we call $\hat{p}_j = -\,i\frac{\partial}{\partial q_j}$ the ***momentum*** *operator* along with the ***quantity*** q_i.

The operator $\hat{q}_j = i\frac{\partial}{\partial p_j}$ is called the ***quantity*** *operator*[65] generated by the ***momentum*** p_i.

We repeat that $\hat{q}_i$ and $\hat{p}_j$ complies with (2.71) $[\hat{q}_i, \hat{p}_j] = i\,\delta_{ij}$, $[\hat{q}_i, \hat{q}_j] = 0$, $[\hat{p}_i, \hat{p}_j] = 0$.
Like the Fourier transform (1.81), now by real ***quantities*** q_j for the operator, we write with Dirac's $\langle \text{bra}|, |\text{ket}\rangle$ notation, where $\langle q|\psi\rangle \coloneqq \psi(q)$, in this way;

(2.82) $$\langle q_j|\hat{p}_j|\psi\rangle = \langle q_j| -i\frac{\partial}{\partial q_j}\psi\rangle = -i\frac{\partial}{\partial q_j}\psi(q_j), \qquad \text{and as an integral}$$

(2.83) $$\langle q_j|\hat{p}_j|\psi\rangle \sim \int q_j^*\left(-i\frac{\partial}{\partial q_j}\psi\right)dq_j,$$

with the classical association $\tilde{p}_j(\omega) = \frac{1}{2\pi}\int_{\mathbb{R}} dt e^{it\cdot\omega} q_j(t)$.
And similar to the inverse Fourier transform (1.80) we now define the ***quantity*** *operator* $\hat{q}_j$ from the ***momentum*** p_j

(2.84) $$\langle p_j|\hat{q}_j|\psi\rangle = i\frac{\partial}{\partial p_j}\psi(p_j) \sim \int p_j^*\left(i\frac{\partial\psi}{\partial p_j}\right)dp_j,$$

with the classical association $q_j(t) = \int_{\mathbb{R}} d\omega e^{-i\omega\cdot t}\tilde{p}_j(\omega)$.

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[65] When we talk about a *spatial* ***quantity***, we can call it the *position operator*.