(1.82)
$$q(t') = \frac{1}{2\pi}\int_{\mathbb{R}} e^{i\omega t'}\int_{\vec{\mathbb{R}}} e^{-i\omega t}\, q(t)dt\, d\omega$$
$$= \frac{1}{2\pi}\int_{\vec{\mathbb{R}}}\int_{\mathbb{R}}\left(e^{i(t'-t)\cdot\omega}\right)d\omega \cdot q(t)dt = \int_{\vec{\mathbb{R}}} \delta(t'-t)\cdot q(t)dt,$$

where the last rewriting uses the Dirac delta function given by

(1.83)
$$\delta(t'-t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega(t'-t)}\, d\omega = \frac{1}{2\pi}\int_{\mathbb{R}} e^{i\omega(t'-t)}\, d\omega = \begin{cases} 1 & \text{for } t' = t \\ 0 & \text{for } t' \neq t \end{cases}$$

$q(t)$ only contributes to the integral $\int_{\vec{\mathbb{R}}} \delta(t'-t)\cdot q(t)dt$, when $t = t'$.
The Dirac delta function indeed shows that the development parameter does not carry any ***causality***.
Each t is completely independent of each other t', unless $t' = t$. <u>We say that $t \neq t'$ are orthogonal</u>.

1.7.8. The Local Internal Time

In this review on Fourier *spectral resolution*, we may have done some cheating, because we have provided the same internal parameter $t \in \vec{\mathbb{R}}$ in all the oscillators $u_\omega = e^{-i\omega t}$.
Within the ***entity*** Ψ, we shall choose a local oscillator Ψ_{ω_c}, which angular reference frequency ***quantity*** $\omega_c \in \mathbb{R}$ is measured relative to a local external clock Ψ_{clock} with the defining angular frequency $\omega_{c1} \equiv 1$. Then this clock $u_{\text{clock}} = e^{-i\omega_{c1}t'} = e^{-it'}$ given $\omega_{c1} \equiv 1$ shall be well defined and therefore stable and reliable. It gives the local external information development parameter t' as its own phase, together with its unit measure $[\omega_{c1}^{-1}] \sim \frac{1}{\omega_{c1}} = \frac{T_{c1}}{2\pi} = 1$ for each angular radian.
Hence the full periodic clock frequency is $f_{c1} = \frac{1}{2\pi}$, with only one periodic tuck for every $T_{c1} = 2\pi$.
For the external development parameter synchronisation see § 1.6.2.4 (1.42) and Figure 1.3.

For the chosen local oscillator Ψ_{ω_c} in the ***spectrum*** of ***entity*** Ψ, we measure the angular reference frequency ***quantity*** $\omega_c[\omega_{c1}]$ by the external local reference clock $\Psi_{\text{clock}}^{\omega_{c1}\equiv 1}$.
The phase angle ϕ_c of this reference oscillation $u_{\omega_c} = e^{-i\omega_c t'} = e^{-i\phi_c}$ in the spectrum can be used as the motor for external information development parameter $t' = t_c' = \phi_c/\omega_c$ or the measure

(1.84)
$$t'[\omega_{c1}^{-1}] = \frac{\phi_c}{\omega_c[\omega_{c1}]}$$

This now-established *external development parameter* will, due to (1.83) $(t = t')$ be used in the inverse oscillator $u_{\omega_c}^* = e^{i\omega_c t}$ as an internal information development parameter $t = \phi_c/\omega_c$.
All the other inverse oscillators $u_\omega^* = e^{i\omega t}$ at angular frequencies $\forall\omega \in \mathbb{R}$ in the spectrum can be defined relative to the reference $\omega_c[\omega_{c1}]$. I.e., $\omega = \omega[\omega_{c1}] = \left(\frac{\omega t}{\phi_c}\right)\omega_c[\omega_{c1}]$.

When it is possible to locate a ***spectrum*** for an ***entity*** Ψ and choose a specified local reference oscillator Ψ_{ω_c} at the angular frequency $\omega_c[\omega_{c1}]$ and use its development parameter t_c for all the oscillators in the spectrum, we will call the ***entity*** Ψ *locally definable* with a homogeneous continuous information development parameter $t = t' = t_c \in \vec{\mathbb{R}}$.
From all this (1.80)-(1.83) we conclude; that the ***quantitative*** function $q(t')$, with the argument from the external information development parameter (1.84) $t' = t \in \vec{\mathbb{R}}$, must be able to be locally integrated, and the local external parameter for the ***entity*** Ψ_{ω_c} must be locally synchronous with the internal parameter for all oscillators Ψ_ω in the ***entity*** Ψ.
The conclusion is: there are two aspects of ***quantities*** for a physical ***entity*** Ψ consisting of oscillator ***entities*** Ψ_ω based on the circular rotation concept as a ***primary quality***:

- The ***quantity*** $q(t')$ as a function of the external parameter $t' \in \vec{\mathbb{R}}$ without any causal interaction.
- The ***spectrum*** $\tilde{q}(\omega)$ as a function of the frequency ω, where $\omega \in \mathbb{R}$ is the given[47] ***quantity***.

[47] As long the frequency ω is concerned in the concept of ***entity*** $\Psi_\omega \in \Psi$, it is steady as a given conserved ***quantity***.
This is not a law of physics but is a priori given ***quality*** by our synthetic judgement as our concept of a frequency ***quantity***.

1.7.8.2. The Orthogonal Frequencies

Employing Fourier transformations, we look at the ***quantitative*** angularly frequency ***spectrum*** of the physical ***entity*** Ψ at different angular frequencies

(1.85) $\tilde{q}(\omega') = \mathfrak{F}\{q(t_c), \omega'\} = \frac{1}{2\pi}\int_{\vec{\mathbb{R}}} q(t_c)e^{-i\omega' t_c}\, dt_c,$ and

(1.86) $q(t_c) = \mathfrak{F}^{-1}\{\tilde{q}(\omega), t_c\} = \int_{\mathbb{R}} \tilde{q}(\omega)\, e^{i\omega\, t_c}\, d\omega.$

We compare two different frequencies by the composition $\mathfrak{F}(\mathfrak{F}^{-1})$

(1.87)
$$\tilde{q}(\omega') = \frac{1}{2\pi}\int_{\vec{\mathbb{R}}} e^{-i\omega' t_c}\int_{\mathbb{R}} \tilde{q}(\omega)e^{i\omega\, t_c}\, d\omega\; dt_c$$
$$= \frac{1}{2\pi}\int_{\mathbb{R}}\int_{\vec{\mathbb{R}}} e^{i(\omega-\omega')\cdot t_c}\, dt \cdot \tilde{q}(\omega)d\omega = \int_{\mathbb{R}} \delta(\omega-\omega')\cdot\tilde{q}(\omega)d\omega,$$

and use the Dirac delta function to show that two oscillators are independent.

(1.88)
$$\delta(\omega-\omega') = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i(\omega-\omega')t}\, dt = \frac{1}{2\pi}\int_{\vec{\mathbb{R}}} e^{i(\omega-\omega')t}\, dt = \begin{cases} 1 & \text{for } \omega' = \omega \\ 0 & \text{for } \omega' \neq \omega \end{cases}$$

We say that all the oscillators are mutually orthogonal unless they have the same frequency $\omega' = \omega$.

1.7.8.3. The local Homogeneous Parameter and the Constant Oscillator Frequencies

For the oscillator Ψ_ω the ***quantity*** ω is given constant relative to a local reference clock Ψ_{ω_c}, and the parameter $t \in \vec{\mathbb{R}}$ can be considered as common to all oscillators $\Psi_\omega \in \Psi$ in the ***spectrum*** within a locality in physics. Hereby the one parameter $t \in \vec{\mathbb{R}}$ is defined as homogeneous for the entire local ***spectral entity*** Ψ.

1.7.8.4. Orthogonality and Dependency in the Information Problem

We have now seen that oscillations of different ***quantities*** $\omega \neq \omega'$ are *orthogonal independent*.
The corresponding information parameter measure differentiation $t \neq t'$ are *orthogonal* too, in that, there is *no causality* given from a paraphysical world of one pretended parameter.
After the analysis above it is now obvious that there is an interdependency between these two ***qualities***: the oscillations, and its information measure. We define:

(1.89) $\Delta\omega = \omega' - \omega,$ *and* $\Delta t = t' - t\,.$

We take the product of these two real scalars and postulate the *finite* impact of *interdependency* through the phase angle $\phi = \omega t$ of the oscillator $u_\omega^* = e^{i\omega t}$ expressed in the unit magnitude

(1.90) $0 < \quad |\Delta\omega \cdot \Delta t| = 1 \quad < \infty,$

when they have the same measure foundation for the information $\Delta\omega[s^{-1}]$ and $\Delta t[s]$, hence

(1.91) $[s^{-1}]\cdot[s] = 1.$

When we measure in different unit systems. e.g., frequency energy in electron volt $\Delta\omega$[eV] we define an information measure constant $\hbar$ [eVs] for the equation

(1.92) $|\Delta\omega \cdot \Delta t| = \hbar 1,$ [eV]·[s].

This measure unit scaling constant is called the reduced Planck constant $\hbar = 6.5819\ 10^{-16}$ [eVs].

When it comes to the Heisenberg uncertainty for the measure in one j out of several dimensions, the information uncertainty limit is often expressed as

(1.93) $\left|\Delta p_j \cdot \Delta q_j\right| \geq \hbar½.$