I.e., maintaining constant ***quantities*** $\tilde{q}_\omega$ and $\dot{\tilde{q}}_\omega$ the for the ***primary quality*** $U(1)$.
The equation (1.79) $F = \dot{p}_\omega = -m_\omega \omega^2 q_\omega$ is Newton's second law, which expresses the force as an inner ***quantity*** of the oscillator ***entity*** Ψ_ω, and makes the circular path possible.[41]
This internal force depends on the square of ω and the internal inertia ***quantity*** m_ω.[42]
If we try setting the internal inertia $m_\omega = 0$ the impact of the internal momentum and the internal force will disappear from the oscillator, – the oscillator would stop $\omega = 0$, – void.
Since Descartes and Newton, these ***quantities*** have been essential for any changes in physics, therefore, we must always assume that $m_\omega \neq 0$ for any physical entity. –
Claim this is important to any internal existence of any ***entity*** of matter (das Ding an sich).
When we in 20th-century quantum physics say that a photon has no rest mass $m_{\text{photon}} = 0$, we alluded to the external mass, or say, that the photon doesn't carry portable energy.[43]

When I later below treat the quantum harmonic oscillator, the autonomous reference dimensioning of ***quantities*** will be fixed with the factor $\omega m_\omega = 1$. In this way, we achieve to reduce the impedance factor to the pure imaginary unit $-i$. Then back in (1.77) $z(\omega) = -i$.
Still, later below, I also describe the pure quantum circular oscillator whose angular frequency is auto-normed to 1, i.e., ω=1, with the result, that $m_\omega = 1$ as a purely quantum mechanical view.[44]

This was my abstract introduction to the ***primary quality***, which is called a circle oscillator. This unit circle then oscillates with an externally given ***quantity***, *the angular frequency* ω.

[41] In a cyclic process the Kepler elliptical orbit is possible, but here in this substantial ideology we try to prevent the classical ideas of massive point particles and central forces, Therefor we prefer circular symmetry as a founding idea $U(1)$.
[42] A classic example from space is this; the Moon orbit 'circle' around the Earth. The Moon is a clock for us.
A more day-to-day example is the earth rotation on itself. A one-day clock is relative to the sun or the stars.
The internal inertia factor $m_{2\pi\text{day}}$ of the rotating Earth, does not affect the external rest mass m_{Erth} of the Earth essential.
The impact on the rotating Earth may be discussed in another context.
[43] The external spatial momentum of photons we will treat later in this book.
[44] This last m_ω=1 has been a guess by many authors but often without any explanation except it is simple and beautiful.

1.7.7. Fourier Transformation

An ***entity*** Ψ in physics may contain oscillator ***entities*** Ψ_ω with real angle frequencies ω.
For each $\omega \in \mathbb{R}$, there can be $n_\omega \in \mathbb{N}$ oscillators, $n_\omega \Psi_\omega$.
The collection of all oscillators Ψ_ω over $\forall\omega \in \mathbb{R}$ is called a *spectrum*.[45] (*frequency spectrum*)
In the following, we assume that a spectrum ***entity*** Ψ in physics may contain myriads of oscillator ***entities*** Ψ_ω for each $\omega \in \mathbb{R}$. (the myriads are due to a statistical ensemble view.)
Based on the concept of cyclical time we have in section 1.7.5 above defined an oscillator running swing-spin developing ***quantity*** q_ω that in the calculation reality develops by one continuous parameter $t \in \vec{\mathbb{R}}$ as an argument $q^*_\omega(t) = \tilde{q}^*_\omega u^*_\omega = \tilde{q}^*_\omega e^{i\omega t} \in \mathbb{C}$. (1.60)
Each circular oscillator frequency ω is by definition its own generator for its internal information development parameter t. In fact, we can write $q(\omega, t) = q_\omega(t)$, so that is dependent on the given external ***quantity*** $\omega \in \mathbb{R}$ and internal parameter $t \in \vec{\mathbb{R}}$.
For the external view on the spectrum ***entity*** Ψ we shall define an external local development parameter t' connected to a chosen circular oscillator defined as a standard oscillator *clock* with an angular frequency ω_c that produces its own internal development parameter t_c that shall be promoted to the local external information development parameter $t' := t_c$.
The standard clock shall be in the same locality as the spectrum ***entity*** Ψ. (Due to relativity paradox)

First, we ignore the parameter-dependent internal oscillation $e^{-i\omega t}$ in (1.60) and concentrate on the factor $\tilde{q}_\omega \in \mathbb{C}$, which is explicitly independent of the oscillator internal parameter t, $\frac{\partial}{\partial t}\tilde{q}_\omega = 0$.
We look at all the oscillators $n_\omega \Psi_\omega$ at the frequency ω of the ensemble ***entity*** Ψ and establish the ***quantity*** $\tilde{q}(\omega) = n_\omega \tilde{q}^*_\omega$, for $n_\omega \in \mathbb{N}$ of each $\omega \in \mathbb{R}$.
$\tilde{q}(\omega) \in \mathbb{C}$ is the weighted scalar for each ω, which indicates the contribution of the unit oscillators $u^*_\omega = e^{i\omega t}$ representing each Ψ_ω of the full spectrum ***entity*** Ψ, for all angular frequencies $\forall\omega \in \mathbb{R}$.

For the ensemble ***entity*** Ψ, we now look at a total ***quantity*** $q(t')$, which is dependent on one local macroscopic external information development parameter $t' \in \mathbb{R}$, produced from the local standard oscillator clock (ω_c), for the total ensemble.
Employing the famous Fourier integrals, which we may express as a weighted integral of oscillators $e^{i\omega t'}$, for $\omega \in \mathbb{R}$ and for one local selected external information parameter $t' \in \vec{\mathbb{R}}$:

$$q(t') = \mathfrak{F}^{-1}\{t', \tilde{q}(\omega)\} = \int_{-\infty}^{\infty} \tilde{q}(\omega)\, e^{i\omega t'} d\omega = \int_{\mathbb{R}} \tilde{q}(\omega) e^{i\omega t'} d\omega = \int_{\mathbb{R}} d\omega\, e^{i\omega \cdot t'}\, \tilde{q}(\omega) \tag{1.80}$$

where $\mathfrak{F}^{-1} \sim \int_{\mathbb{R}} d\omega\, e^{i\omega \cdot}$ is the inverse Fourier operator.[46]
Here the weight scalar $\tilde{q}(\omega)$ we will express as the integral over the entire internal parameter span $t \in \vec{\mathbb{R}}$ of the inverted unitary oscillators $u_\omega = e^{-i\omega t}$ weighted by the macroscopic ***quantity*** $q(t)$, here augmented by the internal parameter $t \in \vec{\mathbb{R}}$:

$$\tilde{q}(\omega) = \mathfrak{F}\{\omega, q(t)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} q(t)\, e^{-i\omega t}\, dt = \frac{1}{2\pi}\int_{\mathbb{R}} q(t) e^{-i\omega t}\, dt = \frac{1}{2\pi}\int_{\mathbb{R}} dt\, e^{-it\cdot\omega}\, q(t) \tag{1.81}$$

where $\mathfrak{F} \sim \frac{1}{2\pi}\int_{\mathbb{R}} dt e^{-it\cdot}$ is the Fourier operator.

The ***quantity*** $\tilde{q}(\omega)$ is called a ***spectrum*** of the external ***quantity*** $q(t')$ and therefore the ***spectrum*** for the ***entity*** Ψ.
We combine these two integrals combination $\mathfrak{F}^{-1}(\mathfrak{F})$, and get

[45] Here it is debatable whether the spectrum is continuous or discrete as the number of physical entities deemed to be countable in an intuition universe where not all frequencies $\omega \in \mathbb{R}$ are populated, hence $n_\omega = 0$ are allowed.
[46] Blue notation is used here for the concept of an operator, while magenta is used for any arbitrary operand.