We see that the operator $i\frac{\partial}{\partial t}$ is the general orthogonal relationship between the complex ***quantities*** in the **circle oscillator** and thus just as $U(1)$ a ***primary quality*** for this.
By squaring or using the operator twice, we get the second derived by the one parameter

(1.70) $$\frac{\partial^2}{\partial^2 t}(\varphi_\omega) = i\frac{\partial}{\partial t}\left(i\frac{\partial}{\partial t}\right)(\varphi_\omega) = -\omega^2(\varphi_\omega), \quad \text{or} \quad -\frac{\partial^2}{\partial^2 t}(\varphi_\omega) = i\frac{\partial}{\partial t}\left(i\frac{\partial}{\partial t}\right)^*(\varphi_\omega) = \omega^2(\varphi_\omega).$$

Opposite the *quantum mechanical* differential operator in (1.69), the classical differential operator $\frac{\partial}{\partial t}$ in (1.61), and (1.65) correspond to the factor $\pm i\omega\cdot$ as an imaginary multiplication operator on the classical ***quantities*** of the circular oscillation.[38]
(The issue of the plane concept lay in the imaginary unit i as its concept).

1.7.5.2. The Real Rotation

As we have both retrograde q_ω and progressive q^*_ω rotation, we can combine these two to a real numbered oscillation ***quantity*** φ_ω in a way we call a superposition of the rotations in the complex plane

(1.71) $$\varphi_\omega = \tfrac{1}{2}(q_\omega + q^*_\omega) = \tfrac{1}{2}\left(\tilde{q}_\omega e^{-i\omega t} + \tilde{q}^*_\omega e^{i\omega t}\right) = \varphi^*_\omega$$

(1.72) $$\dot{\varphi}_\omega = \tfrac{1}{2}(\dot{q}_\omega + \dot{q}^*_\omega) = \tfrac{i\omega}{2}\left(-\tilde{q}_\omega e^{-i\omega t} + \tilde{q}^*_\omega e^{i\omega t}\right) = \dot{\varphi}^*_\omega$$

(1.73) $$\ddot{\varphi}_\omega = \tfrac{1}{2}(\ddot{q}_\omega + \ddot{q}^*_\omega) = -\tfrac{\omega^2}{2}\left(\tilde{q}_\omega e^{i\omega t} + \tilde{q}^*_\omega e^{-i\omega t}\right) = \ddot{\varphi}^*_\omega = -\omega^2\varphi_\omega$$

This superposition gives $\varphi^* = \varphi,\ \dot{\varphi}^* = \dot{\varphi},\ \ddot{\varphi}^* = \ddot{\varphi}$, as real ***quantities*** $\varphi, \dot{\varphi} \in \mathbb{R}$.
Since we work in the plane for the $U(1)$ idea: $\phi{=}\omega t \longrightarrow e^{\pm i\omega t}$, we must identify the real basis axis $\left(e^{i0}{=}1\right)$ for the one parameter value defined by $\phi_0 = 0$ for the start phase angle of $\phi{=}\omega t \in \mathbb{R}$.
This axis represents the *phase coherent* start creation ***direction*** for q_ω and q^*_ω at $t = 0 \in \vec{\mathbb{R}}$
At that real number basis axis in the rotation plane, we can combine both the constant complex amplitude numbers $\tilde{q}_\omega$ and $\tilde{q}^*_\omega$ to a real amplitude $\tilde{\varphi}_\omega = \frac{1}{2}\left(\tilde{q}_\omega e^{-i0} + \tilde{q}^*_\omega e^{i0}\right) = \frac{1}{2}(\tilde{q}_\omega + \tilde{q}^*_\omega)$,
note that $\tilde{\varphi}^*_\omega = \tilde{\varphi}_\omega \in \mathbb{R}$, and we shall have $\tilde{\varphi}^2_\omega = \tilde{q}_\omega \tilde{q}^*_\omega$.

We can construct real ***quantities*** from (1.60) and (1.61) by the use of this real amplitude $\tilde{\varphi}_\omega \in \mathbb{R}$ and the superposition (1.71) and (1.72) so we can write the real oscillation as

(1.74) $$\varphi_\omega = \tilde{\varphi}_\omega \cos(\omega t), \qquad \dot{\varphi}_\omega = -\omega\cdot\tilde{\varphi}_\omega \sin(\omega t) \qquad \text{and} \qquad \ddot{\varphi}_\omega = -\omega^2\tilde{\varphi}_\omega \cos(\omega t).$$

We shall remember the old implicit idea, that represents cos( ) represent a projection on the real basis axis, and sin( ) represent a projection on the imaginary basis axis in a Cartesian plane (complex plan).
This paraphrase of a traditional (linear) real oscillator function does not help us much in our understanding of the **circle oscillator** since the rotation in a plane is an intrinsic $U(1)$ symmetry as its primary ***quality***.

If you want the real formulation you end up with the special orthogonal rotation group SO(2) of real 2 × 2 matrices (1.54), which indeed does not make things easier.
I prefer an image form of intuition (face) of the ***entity*** Ψ_ω as an internal oscillation that then is a member of the unitary rotation group $U(1){\sim}(\phi{\to}e^{i\phi}) \sim \left\{\forall u_\omega{=}e^{i\omega t} \in \mathbb{C} \,\middle|\, \omega \in \mathbb{R} \wedge \forall t \in \vec{\mathbb{R}}\right\}$.

1.7.5.3. The Internal Oscillation

In reality, it is an advantage to using the complex ***quantities*** formulation

(1.75) $$\boxed{q_\omega = \tilde{q}_\omega u_\omega} = q_\omega(t) = \tilde{q}_\omega e^{-i\omega t} \quad \text{and} \quad \boxed{\dot{q}_\omega = -i\omega\cdot\tilde{q}_\omega u_\omega}.$$

[38] The ± can be discussed by specifying ω as positive or negative or the rotation progressive or retrograde by chirality problem.

for the circle oscillator, the development parameter $t \in \vec{\mathbb{R}}$ is hidden in $[u_\omega]$, and can therefore be called interior and thus may be omitted from the recitals and from the external form for intuition.[39]
We are left with an idea of an ***entity*** Ψ_ω in physics, the **circle oscillator**, a priori given with the ***quantity*** $\omega \in \mathbb{R}$, where the ***primary quality*** is the unitary rotation group $[u_\omega] \in U(1)$.
In (1.69) I plead for ω as a multiplication operator $\omega\cdot$ equals a differential operator $i\frac{\partial}{\partial t}$.
In that way, we can go from the presumed external ***quantity*** $\omega \in \mathbb{R}$ into the internal characteristic changes managed by the operator $i\frac{\partial}{\partial t}$ shaped along one information development parameter.
This operator is an implicit substance that allows us to look at Ψ_ω by intuition as a plane substance with two linearly independent (orthogonal) complex ***quantities*** q_ω and $\dot{q}_\omega$ for the internal whole, or $\tilde{q}_\omega$ and $\tilde{\dot{q}}_\omega$ for the external view.
In a classical interpretation of these complex ***quantities*** we see by (1.69) the mutual dependence is given by a simple parameter differentiation $\frac{\partial}{\partial t}$ that can be associated with multiplication by the orthogonal factor $\pm i\omega\cdot$ for the circular oscillation in a plane given by an external ***quantity*** $\omega \in \mathbb{R}$ for the ***entity*** Ψ_ω.
In this, the dedicated operation by the imaginary unit i represents a substance in the natural space of physics, and I want to look more into this later in this book (through the idea of geometric algebra).
The relationship between an inner development parameter t for ***entities*** $\Psi_\omega \in \Psi$ and an external information development parameter t' as we by intuition will include for a total ***entity*** Ψ for a phenomenon in physics, will be discussed below, in section 1.7.7.

1.7.6. The Oscillator Rotation in Physics

The circle oscillator given by the quantity $\omega \in \mathbb{R}$ has the immediate swing-spin ***quantity*** q_ω, and its linearly independent derivative by the information development called the rate ***quantity*** $\dot{q}_\omega$
If we imagine internal inertia in the change rate $\dot{q}_\omega$ with an internal *inertia factor* m_ω we can define the oscillator *internal momentum* p_ω as a ***quantity*** of the overall change

(1.76) $$p_\omega = p_\omega(t) = m_\omega \dot{q}_\omega = m_\omega \frac{\partial}{\partial t} q_\omega = -i\omega m_\omega q_\omega = -i\omega m_\omega \tilde{q}_\omega e^{-i\omega t}.$$

It is assumed, that the inertia factor m_ω is explicitly independent of the development parameter t in the oscillator $\frac{\partial}{\partial t} m_\omega = 0$. The factor $-i\omega m_\omega$ is called the *impedance* of the oscillator and is often referred to as a function of the angular frequency $z(\omega)$, whence

(1.77) $$p_\omega = z(\omega)\cdot q_\omega .$$

When the impedance[40] is purely imaginary, as

(1.78) $$z(\omega) = -i\omega m_\omega$$

it provides no resistance to oscillation, as (1.76) when $\tilde{q}_\omega \in \mathbb{R}$ is purely real.
When ω is given constant the internal impedance $z(\omega)$ is considered constant $\frac{\partial z}{\partial t} = 0$.
However, the internal momentum p_ω of the rotation also changes as a function of the internal information development parameter

(1.79) $$\dot{p}_\omega = \frac{\partial}{\partial t} p_\omega(t) = \frac{\partial}{\partial t} m_\omega \dot{q}_\omega(t) = m_\omega \ddot{q}_\omega = -m_\omega \omega^2 q_\omega$$

Does the oscillator possess an internal inertia ***quantity*** m_ω, the ***quantity*** $\dot{p}_\omega$ will precisely the internal force to preserve the circular path $q_\omega = \tilde{q}_\omega u_\omega = \tilde{q}_\omega \cdot e^{-i\omega t}$ of the oscillator.

[39] Here we recall that Newton was operating with an external universal infinity moment (now time), and this has imprinted the classical mechanics since, up to Einstein, and even beyond for many applications.
[40] The impedance denotes an entanglement factor of the internal interdependency.