1.5.3. The Imaginary Approach to the Cyclic Circle of Rotation

Having a real angular parameter $\varphi \in \mathbb{R}$ and multiplying it by the imaginary unit i we achieve a pure imaginary input[28] to the exponential function and hereby get the Euler formula for the circle

$$\varphi \longrightarrow i\varphi \longrightarrow$$

(1.27)
$$\begin{aligned} e^{i\varphi} &= 1 + i\varphi + \frac{(i\varphi)^2}{2!} + \frac{(i\varphi)^3}{3!} + \frac{(i\varphi)^4}{4!} + \frac{(i\varphi)^5}{5!} + \frac{(i\varphi)^6}{6!} + \frac{(i\varphi)^7}{7!} + \frac{(i\varphi)^8}{8!} + \cdots \\ &= 1 + i\varphi - \frac{\varphi^2}{2!} - i\frac{\varphi^3}{3!} + \frac{\varphi^4}{4!} + i\frac{\varphi^5}{5!} - \frac{\varphi^6}{6!} - i\frac{\varphi^7}{7!} + \frac{\varphi^8}{8!} + \cdots \\ &= \left(1 - \frac{\varphi^2}{2!} + \frac{\varphi^4}{4!} - \frac{\varphi^6}{6!} + \frac{\varphi^8}{8!} - \cdots\right) + i\left(\varphi - \frac{\varphi^3}{3!} + \frac{\varphi^5}{5!} - \frac{\varphi^7}{7!} + \cdots\right) = \cos\varphi + i\sin\varphi \end{aligned}$$

This power series development of $e^{i\varphi}$ exploits in the second line, that the exponents of the imaginary unit i is

$(i)^0{=}1,\ (i)^1{=}i,\ (i)^2{=}-1,\ (i)^3{=}-i,\ (i)^4{=}1,\ (i)^5{=}i,\ (i)^6{=}-1,\ (i)^7{=}-i,\ (i)^8{=}1, \ldots$

This can also express the geometric exponent of the perpendicular ⊥ (repeated use of ⊥).

$\perp^0 \to 1,\ \perp^1 \to i,\ \perp^2 \to -1,\ \perp^3 \to -i,\ \perp^4 \to 1,\ \perp^5 \to i,\ \perp^6 \to -1,\ \perp^7 \to -i,\ \perp^8 \to 1, \ldots$

The third line of (1.27) is divided into two sections, the first bracket is just the series of $\cos\varphi$ and the second bracket is just the series of $\sin\varphi$.

Hereby the complex formula appears for

(1.28) The Euler unity circle: $e^{i\varphi} = \cos\varphi + i\sin\varphi \in \mathbb{C}$, for every $\varphi \in \mathbb{R}$

This complex function is a circular periodic

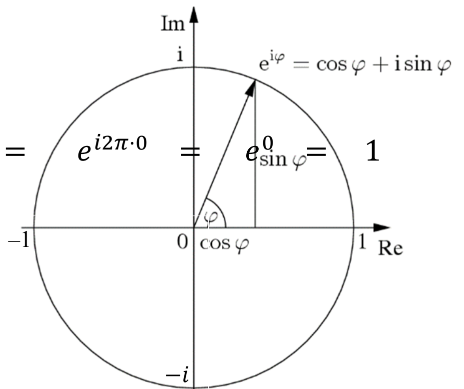
(1.29) $z = e^{i\varphi} = e^{i\varphi + i2\pi n} \in \mathbb{C},\ n \in \mathbb{Z}$ and $\forall\varphi \in \mathbb{R}$

(1.30) One period is divided into four cases: $\perp^0 \ \to\ 1 \ =\ e^{i2\pi\cdot 0} \ =\ e^0 \ =\ 1$

(1.31) $\perp \ \to\ i \ =\ e^{i\frac{2\pi}{4}} \ =\ e^{i\frac{\pi}{2}} \ =\ i$

(1.32) $\perp\cdot\perp \ \to\ i{\cdot}i \ =\ e^{i\frac{\pi}{2}}\cdot e^{i\frac{\pi}{2}} \ =\ e^{i\pi} \ =\ -1$

(1.33) $\perp{\cdot}\perp{\cdot}\perp \ \to\ i{\cdot}i{\cdot}i \ =\ e^{i\frac{3\pi}{2}} \ =\ e^{-i\frac{\pi}{2}} \ =\ -i$



As we see here, the circle is given by $z{=}e^{i\varphi} \in \mathbb{C}$ has four cyclic main events:

The main cases		$z \in \mathbb{C}$			Basis	Axis
Start ↔ Fourth	+ *straight*	$z = +1$	$\varphi = 0$	$\perp^0 = \perp\perp\perp\perp$		*Real*
First	+ *orthogonal*	$z = +i$	$\varphi = \frac{\pi}{2}$	$\perp = -\perp\perp\perp$		*Imaginary*
Second	− *opposite*	$z = -1$	$\varphi = \pi$	$\perp\perp = -\perp\perp$		*Real*
Third	− *orthogonal*	$z = -i$	$\varphi = 3\frac{\pi}{2}$	$\perp\perp\perp = -\perp$		*Imaginary*

Table 1.1: The four main cases of the Euler circle oscillations.

[28] Later in this book chapter II. 5.3 below we will call this argument a pseudoscalar bivector of the plane concept or just a bivector in a geometric multiplication algebra for natural space in physics.

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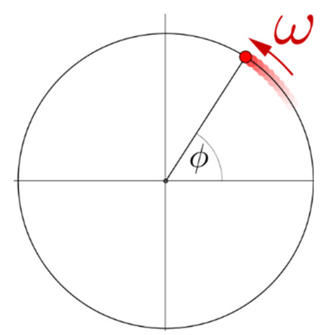
The first and third mode is purely imaginary, while the second and fourth are purely real.
We see an inner causality through the circle of four orthogonal cases from Start (1.30) through First (1.31), Second (1.32), Third (1.33), and Fourth (1.30), which is identical to Start. This creates the inner ***causal action*** as a ***primary quality***, FORWARD. -
This sequential development we call the '*arrow of time*' round the circle in its one plane.
This interior sequential order enables a process that can deliver an identical event, an event that can be counted $n \in \vec{\mathbb{N}}$ for each full turn of the circle $2\pi \leftarrow \perp\perp\perp\perp$. We accumulate $2\pi n$.
If we choose a real running parameter $\tau \in \vec{\mathbb{R}}$ synchronous with n, so that $\lfloor\tau\rfloor{=}n \in \vec{\mathbb{N}}$, this angular passage through the circle can be obtained by the real phase angle $\varphi = 2\pi{\cdot}\tau$
This way (1.28) can be written $e^{i2\pi\cdot\tau} = \cos(2\pi{\cdot}\tau) + i\sin(2\pi{\cdot}\tau)$.
The event itself $e^{i2\pi n} = 1$ for the oscillation is identically repeated and therefore has no causality. Only when identical events are distinguished and counted, do they result in a sequence.
A new event is dependent on the previous and is remembered by the counting number $n \in \vec{\mathbb{N}}$. This we elevate as the running $\tau \in \vec{\mathbb{R}}$ to a real phase angle development parameter $\varphi = 2\pi{\cdot}\tau$ that can be used to distinguish angular events arithmetically in the circle just like the counting numbers $n \in \vec{\mathbb{N}}$ distinguish the full periods.

1.6. The Complex Oscillation - the Circular Movement



We define the angular frequency $\omega \equiv 2\pi f \ \in \mathbb{R}$ from the cyclic oscillation frequency f. Then we can write the actual angle ***quantity*** in the circle as

$$\phi = 2\pi\frac{t}{T} = 2\pi f{\cdot}t = \omega{\cdot}t = \omega t \quad \in \mathbb{R}$$

The parameter $t \in \vec{\mathbb{R}}_{s+}$ goes *FORWARD* and follows the angle ϕ rotating periodically throughout the circle in our model with angular frequency $\omega \in \mathbb{R}$ as the rotary oscillation ***quantity***, which can be either

- Positive $(\omega \geq 0)$ – progressive – right-handed (counterclockwise), or
- Negative $(\omega \leq 0)$ – retrograde – left-handed[29] (clockwise).[30]

The ***quantity*** of circle rotation can also be specified by the oscillation frequency

(1.34) $f = \omega/2\pi$.

The rotating unit circle in terms of the function

(1.35) $e^{i\omega t} = \cos(\omega t) + i\sin(\omega t)$

represents the ***primary quality*** concept idea called a circular oscillating rotation.
We could say that the angle $\phi \in \mathbb{R}$ rotates forward when ω is positive and backward as ω negative, but anyway, the result of the rotation of the circular motion is periodic identical. We have

(1.36) $z = e^{i\phi} = e^{i\phi + i2\pi n} = e^{i\phi}{\cdot}e^{i2\pi n}$, as $e^{i2\pi n} = 1$ for $n \in \mathbb{Z}$.

These same events shall be distinguished and counted irrespective of the orientation of the rotation, e.g., the events $e^{\pm i2\pi n} = 1$ for every $n \in \vec{\mathbb{N}}$.
The rotation $e^{i\omega t} = e^{i2\pi f\cdot t}$, or for the standard clockwise $e^{-i\omega t} = e^{-i2\pi f\cdot t}$ as a ***primary quality*** delivers countable information for each rotation cycle $n \in \vec{\mathbb{N}}$.
The ***quantity*** of the concept of time is again the result of the count, 1, 2, 3, ..., – how far we have been counting times.

[29] These spatial definitions are chiral. The signal is sent from the circle to the observer. (this problem is treated in later sections).
[30] We note, the second-hand display of a clock seen from the face, gives its frequency $f{=}-1$ [oscillation per minute], $(f \equiv \omega/2\pi)$.